THE ELASTIC PROPERTIES OF BISMUTH WIRES.

A THESIS

SUBMITTED TO THE FACULTY OF THE DEPARTMENT OF LITERATURE, SCIENCE AND THE ARTS OF THE UNIVERSITY OF MICHIGAN FOR THE DEGREE OF DOCTOR OF PHILOSOPHY.

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BY J. E. HARRIS.

A MOST striking deviation from the laws of elasticity was first observed a few years ago by Guthe[2] and investigated by him[3] and by Sieg[4] in wires of platinum-iridium alloys. They found that the period of torsional vibration for these wires was by no means constant, but that it decreased rapidly with the amplitude of vibration. The greatest percentage of decrease was found in the smaller amplitudes, the periods seeming to approach a maximum constant value at the larger amplitudes. The logarithmic decrement, starting with the large amplitude, at first increased slowly to a maximum value and then decreased much more rapidly as the amplitude of vibration decreased. The previous treatment of the wire seemed to influence its behavior greatly. After allowing the wire to rest for a long time, or after annealing the wire by heating it to a red heat by means of an electric current, the peculiar properties were not nearly so pronounced as when the wire had just previously been vibrated torsionally for some time. It was also observed that when the wire was kept in vibration for some time at a constant amplitude, the period gradually increased until it finally approached a maximum constant value.

The fact that the previous history and treatment of the wire seemed to influence its behavior to such an extent led to the assumption that the effect was in some way due to the molecular structure of the wire. Since the platinum-iridium alloys are very brittle, it was suggested that perhaps other wires having this same quality of brittleness might show something of the same properties. For this reason, it was decided to investigate the behavior of bismuth wires. This was done and as has been announced by Guthe[5] and the author, the bismuth wires did show the same large decrease in period and logarithmic decrement with decrease in amplitude.

This paper gives the results of the investigations that have been carried on in the past year with bismuth wires.[6]

[1] This investigation was carried out with the aid of a grant from the Elizabeth Thompson Science fund.

[2] K. E. Guthe, Proc. Iowa Academy of Science, 15, p. 147, 1908.

[3] K. E. Guthe and L. P. Sieg, PHYSICAL REVIEW, Vol. 30, No. 4, 1910.

[4] L. P. Sieg, PHYSICAL REVIEW, Vol. 31, No. 4, 1910.

[5] PHYSICAL REVIEW, Vol. 32, p. 228.

[6] The bismuth wires used in these investigations were obtained from Hartman & Braun. They were made from pure, electrolytically deposited bismuth.

II. Apparatus.

The apparatus was similar in most respects to that described by Guthe and Sieg[1] in their investigations with the platinum iridium wires. A large circular scale was used of such dimensions that each degree of amplitude measured about 2 cm. on the circumference of the scale. Each end of the wire was soldered into a brass cylinder. For the purpose of soldering the wires, it was found necessary to mix some bismuth with the solder in order to lower its melting point to a temperature below that for bismuth, the melting point of pure bismuth itself being lower than that of the solder. The upper cylinder was clamped in a device shown in the accompanying diagram. The sketch gives a view at right angles to the axis of the wire. The cylinder to which the upper end of the wire is soldered is held in place by the set-screw *A*. The flat piece of iron *BCDE* carrying the cylinder is free to rotate about the axis of the wire as a center of rotation. By pushing this first one way and then the other, the wire could be made to vibrate. By means of the set-screws *F* and *G*, the angle through which the wire was twisted could be regulated at will. By setting these screws, the wire could also be made, if desired, to vibrate at constant amplitude. The part *HIJK* is a fixed piece of iron and has at one end of it the circular scale *mn*. By means of this scale, the angle through which the wire is being twisted can be determined.

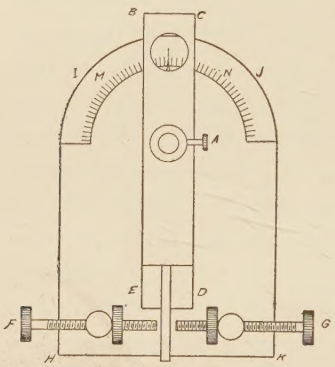

Fig. A.

The cylinder attached to the lower end of the wire was turned in such a way as to leave at its lower end a flat disk 5.58 cm. in diameter. Over the stem part could be slipped cylinders of various masses and diameters, so that the mass and moment of inertia of the vibrating system could be varied as desired. A very thin piece of plane parallel glass, silvered on both sides and fastened vertically to the lower surface of the vibrating disk, served for the reflection of light from an arc lamp to the scale. The lower cylinder was hung in a large beaker made of clear glass, the top of the beaker being covered with a piece of cardboard having a hole in the center through which the wire could pass. This arrangement was made necessary, especially in those observations in which a light suspended weight was used, because it was found that the air currents in the room interfered materially with the observations. The beaker was large

[1] Loc. cit.

enough so as not to introduce any damping effect. A cross-wire placed in a beam of light from an arc lamp was focused on the scale after reflection by the mirror by means of a lens.

The periods of vibration were obtained with the aid of a Morse recorder, one of the electromagnets of the recorder being connected to a relay which in turn was connected to a clock beating seconds, the other electromagnet being connected to the apparatus used for determining the time of the passage of the cross-wire through the point of rest. By measuring the distance between the marks on the tape, an accurate determination of the time of each passage of the cross-wire through the point of rest could be obtained.

In the earlier observations, instead of determining the time of passage of the cross wire through the point of rest by means of a tap-key, a selenium cell was placed in the position occupied by the spot of light reflected by the mirror when the vibrating system was at rest. The cell, which was very sensitive, having a resistance of 198,000 ohms in the dark and only 8,000 ohms in direct sunlight, was placed in series with a two-thousand ohm relay and both connected in a 220-volt circuit. The relay was so adjusted that when the light from the mirror passed over the cell in series with the relay, the decreased resistance of the cell would permit enough current to pass to close the relay, which, in turn closed a circuit containing one of the electromagnets of the recorder, thus giving a mark on the tape. For the larger amplitudes, the entire surface of the cell (about $5 \times 2\frac{1}{2}$ cm.) was exposed to the action of the light. However, as the amplitudes of vibration decreased, the spot of light remained for a continually lengthening period of time on the surface of the cell, so that the mark on the tape could no longer be given at the time of the passage of the spot of light over the middle of the cell but before it reached the middle. For this reason, two pieces of card-board were so arranged that they could be closed in from the ends, thus leaving a continually decreasing portion of the cell exposed to the action of the light. It was found that the slit could be narrowed down in this manner until it was considerably less than a centimeter in width. Even this was too wide for amplitudes less than 20° so that when the amplitude had decreased to this point, the records of the passages of light were obtained by means of a tap-key. Unfortunately, due to the passage of too much current through the cell in one of the observations, the sensitiveness was decreased to such an extent that it could no longer be used, and a tap-key had to be substituted in the observations taken thereafter.

III. METHOD OF PROCEDURE.

The object of the first experiment attempted was to determine the way in which the wire behaved as the initial amplitudes of successive observations were gradually increased. For this purpose, a series of experiments was carried on with two different wires, one of .25 mm. diameter and the other .5 mm. diameter, in each case beginning with an observation in which the initial amplitude was small and increasing the amplitude in each succeeding observation. The amplitudes were increased in each case until the elastic limit of the wire was very nearly reached. For all the experiments, a smooth period amplitude curve could be obtained, but when the curves were compared, no definite relation could be obtained. The curves for experiments, in which the initial amplitude was large, would sometimes be above and sometimes below those in which the initial amplitude was small. This was probably due to the different treatment of the wire on different occasions. It was necessary to adjust the relay in series with the selenium cell as the wire was being vibrated in order to bring it up to the desired initial amplitude and sometimes this adjustment was difficult to bring about, especially if the light from the arc happened to be a little variable. For this reason, it was necessary to keep the wire in vibration much longer in some experiments than in others.

From these results, it became evident that some method of procedure. would have to be found by which more definite results could be obtained In Sieg's[1] work, the method was tried of keeping the wire in constant vibration for some time at the maximum amplitude at which it was desired to begin the experiment and then taking a series of observations as the wire came to rest. It was found that if this experiment was repeated at once without the preliminary treatment, the period amplitude curves for the two sets of readings coincided exactly, indicating that the wire, for the time being at least, had been put in a definite state.

A series of experiments were performed to see if in this respect the bismuth wire behaved like the platinum-iridium wire.

1. *Comparison of Two Series of Observations taken One Immediately After the Other.*—In order to determine definitely the effect of the previous treatment of the wire, two series of observations were taken one immediately after the other. In this experiment, a wire was used that had been used in the experiments mentioned above. The wire had, however, been allowed to rest several days before these experiments were performed. The wire was made to vibrate in each case through an initial amplitude of about 85° and the observations taken as the system

[1] Loc. cit.

came to rest. In this and all other experiments with the bismuth wires, we are limited to small amplitudes compared to those used in the platinum-iridium wires, because of the low elastic limit of the wire. By period in this and in subsequent experiments is meant the period of half a complete vibration. The results for the two experiments are given in Table I., *A* being for the first and *B* for the second series of readings.

TABLE I.

Length of wire, 106 cm. Diam., .5 mm. Moment of inertia, 626 g. $\overline{\text{cm}}^2$. Temp., 23°. Initial amplitude, 86°.3.

No. of Swing.	*A*		*B*	
	Period.	Amp.	Period.	Amp.
5	3.223	73.4	3.224	74.0
10	3.219	63.1	3.221	63.3
15	3.216	54.0	3.217	54.3
20	3.213	46.7	3.215	45.8
25	3.211	40.0	3.213	40.5
30	3.209	34.6	3.210	34.7
35	3.206	30.0	3.207	30.2
40	3.203	26.1	3.205	26.3
45	3.201	22.8	3.203	23.0
50	3.199	19.8	3.201	20.0
55	3.197	17.4	3.200	17.5
60	3.195	15.3	3.198	15.4
65	3.194	13.5	3.196	13.5
70	3.192	11.9	3.194	11.8
75	3.191	10.6	3.193	10.5
80	3.189	9.3	3.192	9.2
85	3.188	8.4	3.190	8.3
90	3.187	7.5	3.189	7.4
95	3.187	6.7	3.188	6.6
100	3.186	6.1	3.187	5.7
105	3.185	5.4	3.187	5.1
110	3.185	4.9	3.186	4.7
115	3.184	4.4	3.185	4.3
120	3.183	3.8	3.185	3.7

The results of this experiment are shown graphically in Fig. 1, the amplitudes being plotted as abscissæ and the periods as ordinates. That the first experiment has had a marked effect on the second is evident from the position of the two curves. The curve for the second experiment lies above that for the first. It will be seen that the second curve in shape is very similar to that of the first. It seems that the effect of the first experiment on the second was merely to displace the curve upward. This effect is due to the so-called "elastic fatigue" in the wire.

2. *Behavior of the Wire when Vibrated at Constant Amplitude.*—In this experiment, several series of observations were taken at different amplitudes to determine whether or not the period remained constant when

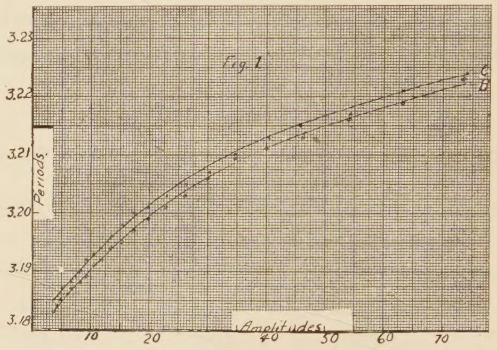

Fig. 1.

the wire was kept vibrating at constant amplitudes. By adjusting the set screws *F* and *G* (see diagram) and moving the lever from one set screw to the other at the end of each swing, the wire could be kept vibrating through any desired amplitude and this amplitude kept constant

TABLE II.

Length of wire = 106 cm. Diam. = .5 mm. Moment of inertia = 626 g. cm^2.

A. Amplitude = 65°.2.

Coincidence.	Intervals.	No. of Vibrations.	Period.
21.21			
53.42	32.21	10	3.221
108.29	87.08	27	3.225
205.13	183.92	57	3.227
After 20 minutes.			
1.56			
85.57	84.01	26	3.231
130.78	129.22	40	3.231
260.07	258.51	80	3.231
B. Amplitude = 93°.5.			
37.70			
79.63	41.93	13	3.225
115.12	77.42	24	3.226
192.61	154.91	48	3.227
After 20 minutes.			
1.54			
62.96	61.42	19	3.233
224.65	223.11	59	3.233
337.82	336.28	104	3.233

within one or two degrees. Only two of the experiments performed are given here for the reason that all of them gave practically the same results. Two series of observations were taken in each case, the first just after the amplitude desired had been reached, and the second, after the wire had been kept in vibration for 20 minutes. The results are shown in Table II.

It will be seen from the above experiments, that the period of vibration increases when the system is made to vibrate at a constant amplitude; that the period of vibration very rapidly approaches a maximum value; and that this maximum value has apparently been reached after an interval of 20 minutes. To make sure of this latter point, the wire was vibrated for twenty minutes, an observation taken and then the wire vibrated for an hour and another observation taken. The period obtained was the same as that obtained after the twenty-minute interval. That a part of the adjustment of the wire takes place while the wire is being vibrated to bring it up to the maximum amplitude is shown by the above tables. In observation *A*, where the wire was brought to its maximum amplitude by larger steps than in *B*, we have a larger variation in the period in the first 50 vibrations than in the corresponding interval in the second observation.

3. *Effect of Continued Vibrations on the Period Amplitude Curve.*—To show this effect, a wire that had not been used before was taken and after allowing it to hang in place for a day, three series of observations were

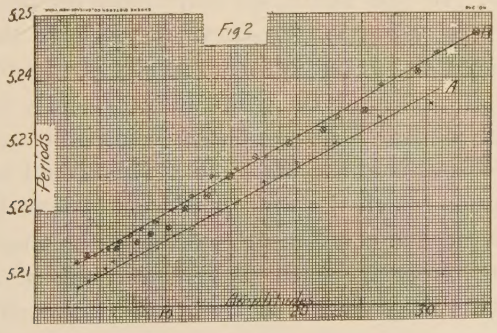

Fig. 2.

taken. In the first, the wire was brought up to the desired amplitude with as few preliminary swings as possible and a series of observations taken as the wire came to rest. In the second experiment, the wire was kept vibrating for a period of about thirty minutes at the same amplitude as that at which the first experiment was begun before taking the series of observations. The third experiment consisted in repeating the first experiment immediately after the second had been completed. The results appear in Table III.

TABLE III.

Length of wire, 103.6 cm. Diam., .25 mm. Moment of inertia, 133 g. $\overline{cm^2}$.

No. of Swing.	Exp. *A*. Initial Amp. 35°.0. Temp. 20° 5.		Exp. *B*. Initial Amp. 35°.4. Temp. 20°.5.		Exp. *C*. Initial Amp. 38°.0. Temp. 20°.8.	
	Period.	Amplitude.	Period.	Amplitude.	Period.	Amplitude.
5	5.236	30.4	5.244	30.8	5.247	33.8
10	5.234	26.3	5.239	26.5	5.241	29.3
15	5.230	22.9	5.234	23.2	5.235	25.4
20	5.227	20.0	5.231	20.1	5.232	22.1
25	5.224	17.5	5.228	17.6	5.230	19.3
30	5.221	15.5	5.226	15.3	5.228	16.9
35	5.219	13.6	5.225	13.5	5.225	14.9
40	5.217	11.9	5.222	11.9	5.222	13.1
45	5.216	10.6	5.220	10.5	5.220	11.5
50	5.214	9.3	5.218	9.2	5.217	10.2
55	5.213	8.3	5.217	8.2	5.216	9.0
60	5.213	7.3	5.216	7.3	5.215	7.9
65	5.213	6.5	5.215	6.5	5.214	7.0
70	5.212	5.9	5.214	5.7	5.214	6.3
75	5.211	5.3	5.213	5.1	5.214	5.6
80	5.210	4.7	5.213	4.6	5.214	5.0
85	5.209	4.1	5.213	4.0	5.213	4.4
90	5.209	3.7	5.213	3.6		
95	5.208	3.3	5.212	3.2		

The results of these experiments are shown graphically in Fig. 2, curves *A* and *B*. In this figure, the amplitudes are plotted as abscissæ and the periods as ordinates. In curve *A*, we have the results of experiment *A* plotted and in curve *C*, the results of *B* and *C*. It will be noticed that in all three cases the points lie very nearly in a straight line. The straight line marked *A* lies somewhat below that marked *B* but the slope of the two lines is about the same. This would indicate that the continued vibration which the wire underwent, before the observations in *B* were taken, has increased the period for all the amplitudes from the maximum to the lowest for which readings were taken. Curve *B* shows that the points for experiments *B* and *C* lie very nearly on the same straight line.

To determine whether or not the results are the same when the wire is kept in continual vibration at a larger amplitude, the experiment was repeated using an initial amplitude of about 190°. This amplitude is slightly under that permitted by the elastic limit of the wire.

The results of these experiments are shown in Fig. 3. The results are quite similar to those found in the preceding experiment. Fig. 3 shows the curve for experiments *B* and *C* lying above that for *A* and the curves for *B* and *C* coinciding. These experiments indicate that the

wire for the time being at least has been put in a definite state. It was determined, as a result of these experiments, that all subsequent series of observations should be taken after the wire had been kept in continuous vibration for a period of from twenty minutes to half an hour.

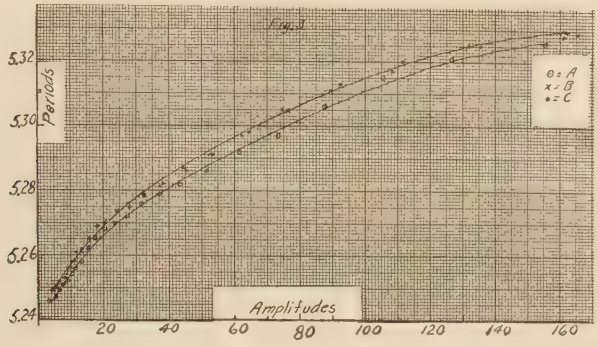

Fig. 3.

TABLE IV.

Length of wire, 106 cm. Diam., .25 mm. Moment of inertia, 133 g. $\overline{\text{cm}}^2$.

No. of Swing.	Exp. *A*. Initial Amp. 184°.8. Temp. 23°.2.		Exp. *B*. Initial Amp. 190°.5. Temp. 23°.6.		Exp. *C*. Initial Amp. 195°.0. Temp. 23°.7.	
	Period.	Amplitude.	Period.	Amplitude.	Period.	Amplitude.
5	5.326	156.5	5.328	161.5	5.329	165.8
10	5.321	127.9	5.325	131.7	5.235	135.8
15	5.315	105.5	5.318	108.2	5.320	111.8
20	5.306	87.6	5.311	89.7	5.313	92.2
25	5.297	73.1	5.305	74.7	5.305	76.8
30	5.292	61.3	5.297	62.2	5.298	64.4
35	5.286	51.6	5.291	52.5	5.291	53.9
40	5.282	43.7	5.287	44.2	5.286	45.7
45	5.279	37.2	5.281	37.6	5.282	38.6
50	5.276	31.8	5.279	32.0	5.278	32.9
55	5.272	27.1	5.276	27.3	5.275	28.1
60	5.270	23.3	5.274	23.6	5.274	24.2
65	5.268	20.3	5.271	20.5	5.270	20.8
70	5.265	17.4	5.269	17.6	5.269	17.9
75	5.262	15.2	5.265	15.3	5.265	15.5
80	5.258	13.2	5.261	13.3	5.262	13.5
85	5.256	11.6	5.258	11.6	5.261	11.8
90	5.254	10.3	5.256	10.1	5.258	10.4
95	5.252	9.0	5.254	8.8	5.256	9.0
100	5.251	7.9	5.253	7.8	5.253	7.9
105	5.249	6.8	5.252	6.8	5.251	7.0
110	5.247	6.0	5.251	6.3	5.250	6.1
115	5.247	5.2	5.250	5.6	5.250	5.3
120	5.246	4.6	5.250	4.9	5.249	4.7
125	5.246	4.1	5.249	4.3		
130	5.246	3.6				

The logarithmic decrements for the various amplitudes were worked out for the above experiment and these logarithmic decrements plotted against the corresponding amplitudes in order to determine the effect of continued vibration on the logarithmic decrement. The results are given in Table V.

TABLE V.

Exp. *A*.			Exp. *B*.			Exp. *C*.		
No. of Swings.	Log. Dec.	Amp.	No. of Swings.	Log. Dec.	Amp.	No. of Swings.	Log. Dec.	Amp.
8	.0178	156.5	8	.0179	161.5	8	.0174	165.8
10	.0170	127.9	10	.0174	131.5	10	.0171	135.8
10	.0166	105.5	10	.0167	108.3	10	.0168	111.8
10	.0160	87.6	10	.0161	89.7	10	.0163	92.2
10	.0155	73.1	10	.0159	74.7	10	.0156	76.8
10	.0151	61.3	10	.0153	62.2	10	.0154	64.4
10	.0147	51.6	10	.0148	52.5	10	.0149	53.9
10	.0142	43.7	10	.0144	44.2	10	.0145	45.7
10	.0138	37.3	10	.0140	37.6	10	.0143	38.6
20	.0137	31.8	20	.0136	32.0	20	.0139	32.9
20	.0132	27.1	20	.0132	26.3	20	.0135	28.1
20	.0128	23.3	20	.0130	23.6	20	.0132	24.2
20	.0126	20.3	20	.0127	20.5	20	.0129	20.9
20	.0123	17.6	20	.0125	17.6	20	.0127	17.9
20	.0122	15.3	20	.0124	15.3	20	.0123	15.5
20	.0116	13.2	20	.0121	13.3	20	.0119	13.5
20	.0114	11.6	20	.0120	11.6	20	.0118	11.8
20	.0111	10.3	20	.0116	10.1	40	.0117	10.4
20	.0115	7.9	40	.0108	7.8	40	.0115	7.9
20	.0113	6.0	40	.0106	5.6	40	.0112	6.1
20	.0113	4.6				40	.0109	4.7
20	.0113	3.6						

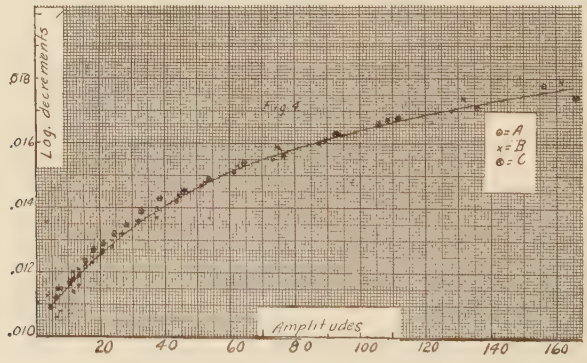

Fig. 4.

In the above table, the first, fourth and seventh columns give the number of swings used in determining the mean logarithmic decrement. The results have been plotted and are shown in Fig. 4. It is seen that

all three sets of points lie very nearly on the same curve, indicating that the continued vibration of the wire has had little if any effect on the logarithmic decrement.

IV. Increasing Amplitudes.

Having settled upon the method of procedure, the first experiment undertaken was to determine the effect of the initial amplitude upon the behavior of the wire. For this purpose, a wire of .25 mm. diameter that had not been used in any experiment before was hung in the bracket with a 50 g. suspension and allowed to hang in position for one day before taking the first series of observations. In the succeeding four days, thereafter, four series of observations were taken, one on each day. On the first, an initial amplitude of 34°.4 was used. On the second the initial amplitude was 84°.2, on the third 131°.0 and on the last 189°.5. In each case the wire was kept vibrating at the initial amplitude for a period of from 20 to 30 minutes before beginning the experiment. The data are contained in Table VI.

Table VI.

Length of wire, 103.6 cm. Diam., .25 mm. Moment of inertia, 133 g. cm².

No. of Swing.	Exp. *A*. Init. Amp. 34°.4. Temp. 20°.5.		Exp. *B*. Init. Amp. 82.°4. Temp. 22°.0.		Exp. *C*. Init. Amp. 131°.0. Temp. 22°.5.		Exp. *D*. Init. Amp. 190°.5. Temp. 23°.6.	
	Per.	Amp.	Per.	Amp.	Per.	Amp.	Per.	Amp.
5	5.244	30.8	5.279	71.9	5.305	112.5	5.328	161.5
10	5.239	26.5	5.272	60.6	5.299	93.3	5.325	131.7
15	5.234	23.2	5.266	51.3	5.291	77.8	5.318	108.2
20	5.231	20.1	5.261	43.7	5.284	65.1	5.311	89.7
25	5.228	17.6	5.256	37.4	5.279	54.8	5.305	74.7
30	5.226	15.3	5.253	32.1	5.273	46.5	5.297	62.2
35	5.225	13.5	5.250	27.6	5.271	39.4	5.291	52.5
40	5.222	13.5	5.247	23.9	5.269	33.7	5.287	44.2
45	5.220	10.5	5.245	20.7	5.263	28.8	5.281	37.6
50	5.218	9.2	5.243	18.0	5.261	24.8	5.279	32.0
55	5.217	8.2	5.241	15.8	5.258	21.3	5.276	27.3
60	5.216	7.3	5.240	13.9	5.254	18.5	5.274	23.6
65	5.215	6.5	5.238	12.0	5.253	16.1	5.271	20.5
70	5.214	5.7	5.236	10.6	5.250	13.9	5.269	17.6
75	5.213	5.1	5.233	9.4	5.248	12.2	5.265	15.3
80	5.213	4.6	5.231	8.3	5.247	10.6	5.261	13.3
85	5.213	3.6	5.229	7.3	5.245	9.4	5.258	11.6
90	5.213	3.6	5.227	6.5	5.244	8.2	5.256	10.1
95	5.212	3.2	5.227	5.7	5.242	7.2	5.254	8.8
100			5.226	5.1	5.240	6.4	5.253	7.8
105			5.225	4.5	5.238	5.7	5.252	6.8
110			5.225	4.0	5.237	5.0	5.251	6.2
115			5.224	3.6			5.250	5.6
120							5.250	4.9
125							5.249	4.3

In Fig. 5 the curves *A*, *B*, *C* and *D* are the period-amplitude curves for experiments *A*, *B*, *C* and *D*. It will be observed that the periods in each successive curve are higher than the corresponding periods of the next preceding curve. These curves are in many respects different from those obtained by Guthe and Sieg with the platinum-iridium wires. In the corresponding experiment with the platinum-iridium wires, while

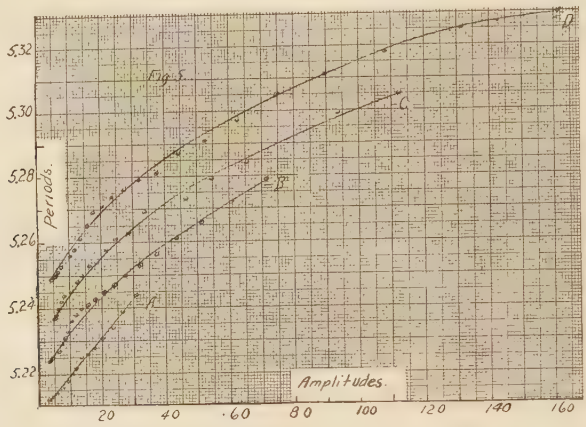

Fig. 5.

the curves for the experiments performed with the large initial amplitudes lay above those for the small amplitudes, the curves seemed to start from a common point, which is not the case with the curves for the bismuth wires. It was also noticed in the experiments with the platinum-iridium wires that at large amplitudes the periods approached a certain maximum value. In this experiment and in all others with the bismuth wires, it was found that, while the curves became less steep with larger amplitudes, no maximum was reached. However, in the case of the bismuth wires, the observations are limited by the low elastic limit to amplitudes that are very low compared to those used with the platinum-iridium wires, so that it may be that the curves shown in the figure correspond to those portions of the curves for the platinum-iridium wires, where the rate of increase in the period is greatest. Also it must be remembered that in the experiment with the platinum-iridium wires the wires were not kept in continuous vibration before taking the observations.

It will be noticed that the curves for the bismuth wires have very much the same shape and slope. The chief difference in the successive curves seems to be one of displacement, the direction of displacement being upward for the larger amplitudes.

The Logarithmic Decrement.—The logarithmic decrements for these

observations were computed and plotted as ordinates with the corresponding amplitudes as abscissæ. The results are given in Table VII. and the curves shown in Fig. 6. The curves A, B, C and D correspond to experiments A, B, C and D.

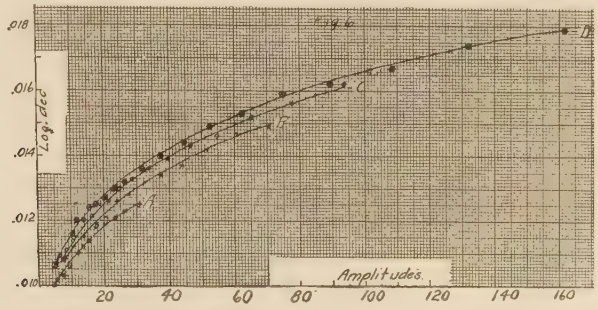

Fig. 6.

It is found that in all the curves there is a marked decrease in the logarithmic decrement with decrease in amplitude. Here, as in the period amplitude curves, we find that each successive curve lies above the next preceding one. These results would indicate that the greater twist of the wire through the large amplitudes increases the internal friction and that this increase in internal friction is not confined to the swings of larger amplitude but persists through the smaller amplitudes.

In the case of the logarithmic decrement-amplitude curves we again notice a striking difference from the corresponding curves for the platinum-iridium wires. In those wires, the logarithmic decrement at first, starting with the large amplitudes, increased slowly to a maximum value and then decreased very rapidly. It may be that here, too, the curve for the bismuth wire corresponds to that portion of the logarithmic decrement curve for the platinum-iridium wires where the amplitudes are small and the values of the decrement are decreasing.

Period and Vibration Number Curve.—For the last of the above four observations, that in which the initial amplitude was greatest, the periods were plotted as ordinates and the corresponding vibration numbers as abscissæ. For lack of space this curve is not shown here. It was found however that this curve also differs from the corresponding curve for the platinum-iridium wire.[1] In the latter case, the curve starts out almost parallel to the x-axis, then bends rapidly downward, and finally again approaches a horizontal direction. Neither in this, nor in any other curve of the same sort for the bismuth wire, could the tendency to start out in a direction parallel to the x-axis be detected. In other words, the curve is convex downward throughout its length. However, here again, it is

[1] Sieg, loc. cit., p. 425.

TABLE VII.

Exp. *A.*			Exp. *B.*		
No. of Swings.	Log. Dec.	Amp.	No. of Swings.	Log. Dec.	Amp.
8	.0125	30.8	8	.0148	70.9
10	.0123	26.5	10	.0147	60.6
20	.0121	23.2	10	.0142	51.3
20	.0119	20.1	10	.0137	43.7
20	.0118	17.6	10	.0134	37.4
20	.0114	15.3	10	.0132	32.1
20	.0112	13.5	20	.0128	27.6
40	.0110	11.9	20	.0126	23.9
40	.0107	9.2	20	.0121	20.7
40	.0103	7.3	20	.0118	18.0
40	.0102	5.7	20	.0115	13.9
40	.0100	4.6	20	.0112	10.6
			40	.0109	8.3
			40	.0106	5.1
Exp. *C.*			Exp. *D.*		
No. of Swings.	Log. Dec.	Amp.	No. of Swings.	Log. Dec.	Amp.
8	.0164	112.5	8	.0179	161.5
10	.0162	93.3	10	.0174	131.7
10	.0156	77.8	10	.0167	108.2
10	.0152	65.1	10	.0161	89.7
10	.0146	54.8	10	.0159	74.7
10	.0143	46.5	10	.0153	62.2
10	.0139	39.4	10	.0148	52.5
10	.0136	33.7	10	.0144	44.2
10	.0133	28.8	10	.0140	37.6
20	.0130	24.8	20	.0136	32.0
20	.0126	21.3	20	.0132	26.3
20	.0125	18.5	20	.0130	23.6
20	.0121	16.1	20	.0127	20.5
40	.0120	13.9	20	.0125	17.6
40	.0115	10.6	20	.0124	15.3
40	.0111	8.2	20	.0121	13.3
40	.0109	6.4	20	.0120	11.6
			20	.0116	10.1
			40	.0108	7.8
			40	.0106	5.6

that portion of the curve for the platinum-iridium wires that corresponds to the higher amplitudes that is missing in the curves for the bismuth wires.

In one other respect does the action of the bismuth wire differ from the platinum-iridium wire. In the case of the platinum-iridium wires, it was found that the wire made a larger number of vibrations in coming to rest when started with small amplitudes than when started with

larger amplitudes. Thus when the platinum-iridium wire was started with an initial amplitude of 364.3 degrees, it required 125 swings to fall to 5 degrees, while, when the wire was started with an initial amplitude of 14.3 degrees, 200 swings were required for the amplitude to fall to 6.15 degrees.[1] This same effect can be noted in the bismuth wires but not in anything like the same degree. In experiment *A* above, in which the initial amplitude was 34°.4, it required 89 vibrations for the amplitude to fall to 3.6 degrees. In experiment *D* in which the initial amplitude was 190.5 degrees, it required only 80 vibrations to cover the same interval.

V. Effect of Changing the Moment of Inertia.

The next experiment undertaken was to determine the effect of changing the moment of inertia of the vibrating system on the period-amplitude curve and on the logarithmic decrement curve. For this purpose, a wire .5 mm. in diameter and 95.7 cm. in length was used. Three experiments

Table VIII.

Diam. of wire, .5 mm. Length, 95.7 cm.

No. of Swing.	Exp. *A*. Mom. of Iner. 1,299 g. cm^2. Mass 150 g. Initial Amp. 89°.4. Temp. 26°.0.		Exp. *B*. Mom. of Iner. 1,882 g. cm^2. Mass 200 g. Initial Amp. 86°.5. Temp. 26°.0.		Exp. *C*. Mom. of Iner. 1,456 g. cm^2. Mass 200 g. Initial Amp. 82°.9. Temp. 25°.8.	
	Period.	Amplitude.	Period.	Amplitude.	Period.	Amplitude.
5	4.214	77.1	5.075	74.2	4.471	71.3
10	4.207	64.2	5.068	61.8	4.464	59.4
15	4.200	54.0	5.062	51.6	4.459	49.6
20	4.193	45.3	5.054	43.3	4.453	41.6
25	4.186	38.2	5.048	36.5	4.449	35.1
30	4.181	33.3	5.043	30.8	4.445	29.8
35	4.177	27.5	5.037	26.1	4.441	25.2
40	4.174	23.5	5.033	22.1	4.437	21.7
45	4.171	19.9	5.029	19.0	4.433	18.4
50	4.168	17.2	5.026	16.3	4.428	15.7
55	4.165	14.8	5.024	14.0	4.425	13.5
60	4.162	12.7	5.022	12.1	4.423	11.8
65	4.160	10.9	5.019	10.5	4.419	10.2
70	4.158	9.5	5.015	9.1	4.417	8.9
75	4.156	8.1	5.012	8.0	4.415	7.8
80	4.155	7.1	5.010	6.9	4.413	6.8
85	4.153	6.2	5.006	6.1	4.410	5.9
90	4.151	5.5	5.004	5.3	4.409	5.2
95	4.150	4.8	5.002	4.7	4.407	4.2
100	4.148	4.2	5.000	4.1	4.405	3.9
105	4.147	3.7	5.000	3.7		
110	4.147	3.2				

[1] Guthe and Sieg, loc. cit., p. 617.

were carried out, A, with a suspended weight having a mass of 150 g. and a moment of inertia of 1,299 g. $\overline{cm}^2$; B, with a weight of 200 g. and a moment of inertia of 1,882 g. $\overline{cm}^2$; and C, with a weight of 200 g. and a moment of inertia of 1,456 g. $\overline{cm}^2$. By comparing experiments A and B, it was possible to determine the effect of a change in both the moment of inertia and the mass of the suspended weight. By comparing B and C, it was possible to determine the effect of changing the moment of inertia only. The results are shown in Table VIII.

The period-amplitude curves are of the same general appearance as those found in the experiments described above. In these experiments, it was found necessary to use smaller amplitudes since the wire had a lower elastic limit than the one of smaller cross-sectional area used in the earlier experiments.

TABLE IX.

B factor = .8813. A factor = 1.0615.

Old Period.	New Period.	Amplitude.	Old Period.	New Period.	Amplitude.
5.975	4.473	74.2	4.214	4.473	77.1
5.068	4.466	61.8	4.207	4.465	64.2
5.062	4.461	51.6	4.200	4.458	54.0
5.054	4.454	43.3	4.193	4.451	45.3
5.048	4.449	36.5	4.186	4.443	38.1
5.043	4.444	30.8	4.181	4.438	33.4
5.037	4.439	26.1	4.177	4.434	27.5
5.033	4.436	22.1	4.174	4.430	23.5
5.029	4.432	19.0	4.171	4.427	19.9
5.026	4.429	16.3	4.168	4.424	17.1
5.024	4.428	14.0	4.165	4.421	14.8
5.022	4.426	12.2	4.162	4.418	12.7
5.019	4.423	10.3	4.160	4.416	10.9
5.015	4.420	9.1	4.158	4.413	9.5
5.012	4.417	8.0	4.156	4.411	8.1
5.010	4.415	6.9	4.155	4.410	7.1
5.006	4.412	6.1	4.153	4.408	6.2
5.004	4.410	5.3	4.151	4.406	5.5
5.002	4.408	4.7	4.150	4.405	4.8
5.000	4.407	4.2	4.148	4.404	4.2
5.000	4.407	3.7	4.147	4.402	3.7
			4.147	4.402	3.2

In order to afford a means of comparison for the three curves, the three series of observations were all reduced to the same period, that of experiment C. To do this, the three curves were produced until they cut the y-axis and from these points of intersection, a reduction factor was obtained. All the values for the periods of the curve to be reduced were then multiplied by this factor. This operation will have the effect of making the two curves to be compared start with the same period at

zero amplitude. In Table IX. are shown the results for experiments A and B reduced in the manner described above so as to correspond to that of C.

These reduced values were plotted and the curves are shown in Fig. 7. Curve A represents the results for experiment A and curve C the results for experiments B and C. It is found from Fig. 7, that the curve for

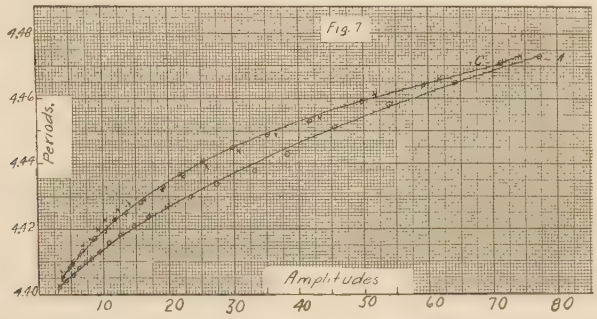

Fig. 7.

experiment A starts and finishes at about the same places as that for experiment C, but the middle portion lies quite a little below that for C. In other words, the curve for A is much flatter than that for C. On the other hand, the curves for B and C very nearly coincide. In fact, the variation in position of the two sets of points in the combined curve is no greater than the variation in the position of the points in either curve considered alone. When we remember that in observations A and C, both the mass of the vibrating system and its moment of inertia were varied, while in B and C only the moment of inertia was varied, we must conclude that, if we keep the stress to which the wire is subjected constant, the period will be the same function of the amplitude no matter what the moment of inertia, while if the stress as well as the moment of inertia is varied, the period will be a different function of the amplitude.

From the formula for the period, $T = 2\pi\sqrt{I/T}$, we should expect the reduction factor in each case to be equal to the square root of the ratio of the corresponding moments of inertia. The reduction factor for reducing observation A to observation C is 1.061. The value of $\sqrt{I_A/I_B}$ is 1.059. The reduction factor for reducing C to B is .881 while the value of $\sqrt{I_B/I_C} = .879$. When we remember that the factors here given were obtained from the curves, the agreement is remarkably close.

For lack of space the curves showing the relation between the logarithmic decrements and amplitudes in this experiment are not shown but it was found that here again we have a very good agreement between the curves for experiments B and C while the curve for experiment A differs to quite a marked extent from the other two, the curve for experiment A

lying below that for experiments *B* and *C*. This difference is especially noticeable in that portion of the curve corresponding to the larger amplitudes. This indicates that the shape of the logarithmic decrement-amplitude curve is also independent of the moment of inertia of the suspended weight but does depend upon the stress to which the wire is subjected. The effect of increasing the mass of the vibrating system is to increase the values for the logarithmic decrement especially for the higher amplitudes.

VI. The Effect of Variation in the Length of the Wire.

For determining the effect of a variation in the length of the wire upon the period-amplitude curve, a wire .25 mm. in diameter was used. Observations were taken using three different lengths: the first 104.8 cm., the second 82.0 cm. and the third 62.6 cm.

In all three cases, a cylinder weighing 50 g. and having a moment of inertia of 133 g. $\overline{cm}^2$. was used. The results obtained are given in Table X.

Table X.

Diam., .25 mm. Moment of inertia, 133 g. $\overline{cm}^2$.

No. of Swing.	Exp. *A*. Length 104.8 cm. Initial Amp. 175°.4. Temp. 26°.0.		Exp. *B*. Length 84.0 cm. Initial Amp. 136°.9. Temp. 26°.5.		Exp. *C*. Length 62.6 cm. Initial Amp. 105°.3. Temp. 27°.0.	
	Period.	Amplitude.	Period.	Amplitude.	Period.	Amplitude.
5	5.306	149.1	4.734	116.1	4.131	88.8
10	5.296	121.5	4.725	94.5	4.126	72.3
15	5.290	100.0	4.717	77.2	4.120	59.6
20	5.283	82.6	4.710	63.7	4.115	49.2
25	5.277	68.7	4.704	52.7	4.109	40.8
30	5.271	57.2	4.699	43.9	4.104	34.1
35	5.265	47.9	4.695	36.6	4.100	28.5
40	5.260	40.3	4.690	30.7	4.097	24.0
45	5.256	33.9	4.686	25.8	4.094	20.2
50	5.252	28.9	4.682	21.9	4.091	17.3
55	5.248	24.5	4.679	18.5	4.089	14.7
60	5.245	21.0	4.675	15.8	4.085	12.6
65	5.242	17.8	4.673	13.6	4.082	10.8
70	5.239	15.4	4.670	11.6	4.080	9.2
75	5.237	13.3	4.668	9.9	4.078	8.1
80	5.235	11.5	4.667	8.6	4.076	7.0
85	5.233	9.9	4.666	7.4	4.075	6.1
90	5.232	8.6	4.664	6.5	4.073	5.3
95	5.231	7.5	4.663	5.6	4.072	4.5
100	5.230	6.5	4.662	4.9	4.070	4.0
105	5.230	5.6	4.661	4.3	4.069	3.5
110	5.229	5.0			4.068	3.1
115	5.228	4.4			4.067	2.7
120	5.227	3.8				

In this case, as in the determination of the effect of a change in moment of inertia, it is necessary to reduce all experiments to the same period in order to obtain a basis for comparison. Experiment *B* was chosen as the basis for the comparison and the periods in the other two series reduced to this. The factors for reduction were obtained in the same way as before, that is, by determining the points of intersection of the curves with the *y*-axis. In this way, the curves will all start with the same period at zero amplitude. The reduced values for experiments *A* and *C* are given in Table XI.

TABLE XI.

A factor = .8912. *C* factor = 1.1458.

Old Period.	New Period.	Amplitude.	Old Period.	New Period.	Amplitude.
5.306	4.729	149.1	4.131	4.733	88.8
5.296	4.720	121.5	4.126	4.727	72.5
5.290	4.715	100.0	4.120	4.720	59.6
5.283	4.708	82.6	4.115	4.715	49.2
5.277	4.703	68.7	4.109	4.708	40.8
5.271	4.698	57.2	4.104	4.702	34.1
5.265	4.692	47.9	4.100	4.698	28.5
5.260	4.688	40.3	4.097	4.694	24.0
5.256	4.684	33.9	4.094	4.691	20.2
5.252	4.680	28.9	4.091	4.687	17.3
5.248	4.677	24.5	4.089	4.684	14.7
5.245	4.674	21.0	4.085	4.680	12.6
5.242	4.672	17.8	4.082	4.677	10.8
5.239	4.669	15.4	4.080	4.675	9.2
5.237	4.667	13.3	4.078	4.672	8.1
5.235	4.665	11.5	4.076	4.670	7.0
5.233	4.664	9.9	4.075	4.669	6.1
5.232	4.663	8.6	4.073	4.667	5.3
5.231	4.662	7.5	4.072	4.665	4.5
5.230	4.661	6.5	4.070	4.663	4.0
5.230	4.661	5.6	4.069	4.662	3.5
5.229	4.660	5.0	4.068	4.661	3.1
5.228	4.659	4.4	4.067	4.660	2.7
5.227	4.658	3.8			

In reducing the results of experiments *A* and *C* to compare with those of *B*, we should expect the factor in each case to be equal to the ratio of the square roots of the lengths, since the period of vibration varies as the square root of the length. The factor for the reduction of the data of experiment *A* to those of experiment *B* is. 8912. The value $\sqrt{L_B/L_A}$ is .8846, giving a difference of .0066. The factor for the reduction of *C* to *B* is 1.1458. The value of $\sqrt{L_B/L_A} = 1.1445$, giving a difference of .0013.

For lack of space the curves for these results are not shown but it was found that the curves did not coincide in this case as they did in the moment of inertia experiment. The curves for the shorter lengths were found to be much steeper than for the longer length. The amounts of decrease in period were about the same in all three cases. It must be remembered, however, that in this experiment we are not comparing corresponding points in the wire. That is to say, the point whose arc of vibration we are measuring in experiment *C* would vibrate through a much larger arc for the same amount of twist in the wire, if the point were at the end of a longer wire such as we have in experiment *A*. For this reason, a reduction factor must be applied to the amplitudes as well as to the period. These reduction factors were obtained by comparing the amplitudes corresponding to the same period in the two curves under consideration. All the amplitudes in the given experiment were then multiplied by this reduction factor. In this reduction, experiment *B* was taken as the standard and the amplitudes of *A* and *C* reduced so as to correspond. The reduced results for experiments *A* and *C* are shown in Table XII.

TABLE XII.

Experiment *A*. Factor = .732.			Experiment *C*. Factor = —.		
Old Amp.	New Amp.	Period.	Old. Amp.	New Amp.	Period.
149.1	110.1	4.729	88.8	128.7	4.733
121.5	88.9	4.720	72.5	105.1	4.727
100.0	73.2	4.715	59.6	86.4	4.720
82.6	60.5	4.708	49.2	71.1	4.715
68.7	50.3	4.703	40.8	59.2	4.708
57.2	42.4	4.698	34.1	49.4	4.702
47.9	35.1	4.692	28.5	41.3	4.698
40.3	29.5	4.688	24.0	34.8	4.694
33.9	24.8	4.684	20.2	29.3	4.691
28.9	21.1	4.680	17.3	25.0	4.687
24.5	17.9	4.677	14.7	21.3	4.684
21.0	15.4	4.674	12.6	18.3	4.680
17.8	13.0	4.672	10.8	15.7	4.677
15.4	11.3	4.669	9.2	13.3	4.675
13.3	9.6	4.667	8.1	11.7	4.672
11.5	8.1	4.665	7.0	10.2	4.670
9.9	7.2	4.664	6.1	8.8	4.669
8.6	6.3	4.663	5.3	7.7	4.667
7.5	5.5	4.662	4.5	6.5	4.665
6.5	4.7	4.661	4.0	5.8	4.663
5.6	4.1	4.661	3.5	5.1	4.662
5.0	3.7	4.660	3.1	4.5	4.661
4.4	3.2	4.659	2.7	3.9	4.660
3.8	2.8	4.658			

The curves obtained from these results together with the original curve for experiment *B* are shown in Fig. 8. We see from that figure that we have a remarkably close agreement between the three sets of points.

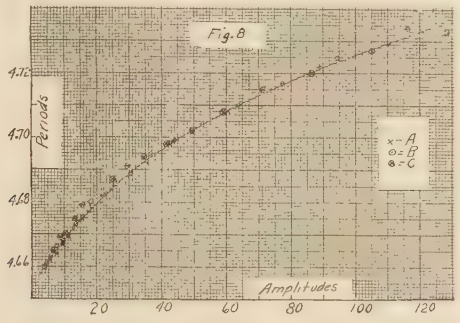

Fig. 8.

If the curves for these experiments were straight lines, we would expect the reduction factors for the amplitudes to be equal to the ratio of the lengths. As it is they are quite different. The reduction factor for changing *A* to *B* is .732 while the ratio of the lengths is .782. The reduction factor for the amplitudes of *C* is 1.450, while the ratio of the lengths is 1.310.

We must conclude from the fact that by merely applying constant factors to the two variables in these observations, the three curves can be made to coincide, that the period in each case is the same function of the amplitude. It also indicates that the wire is uniform throughout its length and that its peculiar behavior is not due to any particular portion of the wire.

VII. The Effect of Variation in Diameter on the Period-Amplitude Curve and the Logarithmic Decrement-Amplitude Curve.

To determine the effect of varying the diameter of the wires upon the period-amplitude curve and the logarithmic decrement-amplitude curve, two wires of different diameter were used with suspended weights of such a mass that the stress to which the wire was subjected was the same in both cases.

In this experiment, the wires were of .25 mm. and .5 mm. in diameter respectively. Since the cross-sectional area of the first was one fourth that of the second, the mass of the suspended weight in the first case was one fourth that in the second. Masses of 50 g. and 200 g. respectively were used.

TABLE XIII.

No. of Swing.	Exp. *A*. Diam. .25 mm. Length 97.9 cm. Mass 50 g. Moment of Inertia 133 g. cm². Temp. 28°.1.		Exp. *C*. Diam. .5 mm. Length 97.9 cm. Mass 200 g. Moment of Inertia 1,456 g. cm². Temp. 26°.0.	
	Period.	Amplitude.	Period.	Amplitude.
5	5.320	142.4	4.521	72.9
10	5.309	114.7	4.514	60.7
15	5.299	92.7	4.509	50.7
20	5.292	75.6	4.503	42.8
25	5.285	62.1	4.499	35.9
30	5.279	51.2	4.496	30.5
35	5.274	42.4	4.491	25.8
40	5.268	35.3	4.487	22.2
45	5.263	29.6	4.483	18.8
50	5.259	24.9	4.478	16.1
55	5.256	21.2	4.475	13.8
60	5.253	17.9	4.472	12.1
65	5.251	15.4	4.468	10.4
70	5.249	13.1	4.466	9.1
75	5.246	11.3	4.464	8.0
80	5.243	9.3	4.462	7.0
85	5.241	8.4	4.459	6.0
90	5.239	7.2	4.458	5.3
95	5.238	6.2	4.457	4.6
100	5.237	5.4	4.456	4.0
105	5.236	4.8		
110	5.235	4.1		
115	5.235	3.6		
120	5.234	3.2		

In this experiment, it was necessary to use a smaller initial amplitude for the wire of large diameter than for the one of small diameter because of the lower elastic limit in the one case than in the other. Of course,

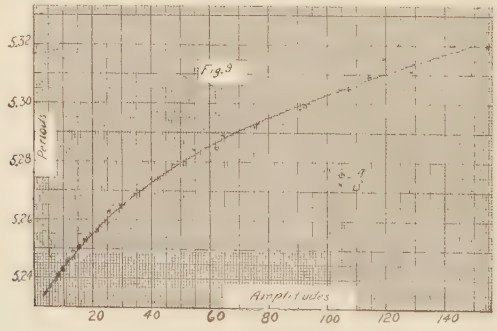

Fig. 9.

here again, we must apply a reduction factor to the periods in order to afford a basis of comparison. This was done, the factor being obtained

in the same way as before. Experiment *A* was taken as the standard and *B* reduced to correspond.

The factor for reducing the period should theoretically be equal to the inverse ratio of the squares of the diameters multiplied by the ratio of the square-roots of the moments of inertia. This operation gives a factor of 1.208, while that found from a comparison of the curves was 1.176.

If a second reduction factor be applied to the amplitudes it is found that the curves coincide. These reduced results are shown in Table XIV. and the two sets of points are plotted in Fig. 9.

TABLE XIV.

Factor = 2.12.

Old Amp.	New Amp.	Period.	Old Amp.	New Amp.	Period.
72.9	154.5	5.320	13.8	29.2	5.265
60.7	128.7	5.311	12.1	25.7	5.263
50.7	107.5	5.305	10.4	22.5	5.258
42.8	90.7	5.298	9.1	19.3	5.255
35.9	76.1	5.293	8.0	17.0	5.253
30.5	64.7	5.289	7.0	14.9	5.250
25.8	54.7	5.284	6.0	13.2	5.247
22.2	47.1	5.279	5.3	11.2	5.245
18.8	39.9	5.274	4.6	9.8	5.243
16.1	34.1	5.269	4.0	8.0	5.241

From the fact that the curves for the two observations can be made to coincide by applying constants to the two variables, we must conclude that when the stress to which the wire is subjected is kept constant, the period is the same general function of the amplitudes no matter what the diameter of the wire.

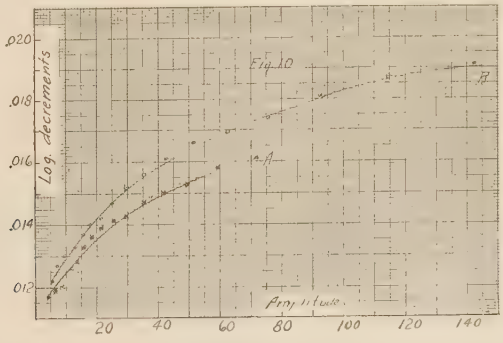

Fig. 10.

The Logarithmic Decrement-Amplitude Curve.—The logarithmic decrements in these two experiments were computed and plotted against the corresponding values of the amplitudes. The results of these computations are shown in Table XV. and the curves in Fig. 10.

TABLE XV.

Exp. *A*.			Exp. *B*.		
No. of Swings.	Log. Dec.	Amplitude.	No. of Swings.	Log. Dec.	Amplitude.
8	.0191	142.2	8	.0161	71.3
10	.0186	114.7	10	.0158	59.4
10	.0181	92.7	10	.0153	49.6
10	.0174	75.6	10	.0150	41.8
10	.0169	62.1	10	.0147	35.1
10	.0166	51.2	20	.0143	29.8
10	.0161	42.4	20	.0141	25.2
10	.0156	35.3	20	.0139	21.7
10	.0152	29.6	20	.0136	18.4
20	.0147	24.9	20	.0133	15.7
20	.0142	21.2	20	.0128	13.5
20	.0140	17.9	40	.0126	11.8
20	.0137	15.4	40	.0120	8.9
20	.0133	13.1	40	.0118	6.8
20	.0130	9.7			
20	.0127	7.2			
20	.0122	5.4			
20	.0117	4.1			

It will be noticed from Fig. 10 that while the logarithmic decrement curves start from about the same point, the one for the small diameter is steeper than that for the wire of larger diameter.

SUMMARY.

1. The elastic properties of bismuth wires have been investigated and it has been found that they display some of the same peculiar properties shown by the platinum-iridium wires investigated by Guthe and Sieg. When the wires were vibrated torsionally, the period of vibration and the logarithmic decrement were found to decrease enormously with the decrease in amplitude. The curves for the bismuth wires representing the relations between (*a*) the period and amplitude, (*b*) the logarithmic decrement and amplitude, and (*c*) the period and vibration number, were found to be lacking in certain characteristics found in the corresponding curves for the platinum-iridium wires. These characteristics were for the most part, however, found at amplitudes for the platinum-iridium wires that could not be reached with the bismuth wires, because of the low elastic limit of the latter.

2. It was found that, in two experiments in which the mass of the vibrating system was kept constant but the moment of inertia varied, the period-amplitude curves in the two cases could be made to coincide by applying a reduction factor to the periods in one set of observations.

This reduction factor was found to be equal to the ratio of the square roots of the moments of inertia used in the two experiments. This indicates that the mathematical relation between the period and amplitude does not depend upon the rapidity of motion of the vibrating system. The logarithmic decrement-amplitude curves in the two cases were found to coincide.

If the load as well as the moment of inertia was varied, it was found that when the proper reduction factor was applied to the periods, the two curves did not coincide, the curve corresponding to the vibrating system having the smaller mass being flatter than the other. The curves for the logarithmic decrement did not coincide, the greater values for the logarithmic decrement being found in the case in which the mass of the suspended weight was greatest.

3. When the length of the wire was varied, it was found that by applying reduction factors to the periods and amplitudes, the period-amplitude curves for the different experiments could be made to coincide, indicating that the period is the same general function of the amplitude in all cases. This also indicates that the wire is uniform throughout its length and that its peculiar behavior is not due to any particular section of the wire.

4. Two wires of different diameter were used with suspended weights of such a mass that the stresses to which the wires were subjected were the same in each case. In this experiment, also, by applying the proper factors to the amplitudes and periods the curves could be made to coincide. The logarithmic decrement curve for the smaller wire was found to lie above that for the wire of larger diameter, indicating a greater damping effect in the former.

In conclusion, the writer wishes to express his appreciation of the helpful advice and kindly interest of Prof. K. E. Guthe, at whose suggestion this investigation was undertaken.

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